

## Recent Advances in Hypersonic Flow Research

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RECENT advances in hypersonic flow research have derived much of their impetus from studies of missiles and spacecraft. As a frame of reference for the subsequent discussion, some of the critical areas in the study of re-entry vehicles will be noted first which are reflected in the current hypersonic flow investigations and may influence research in the future (see, e.g., Refs. 1-5).

Existing theories and experiments have been extremely useful in the solution of a number of engineering problems for atmospheric entry at satellite speed or lower. However, present knowledge of hypersonic flow problems still is far from complete. In addition, at superorbital entry speeds and with maximum heat rates occurring at higher altitudes,<sup>1-3</sup> other phenomena that have not yet received full attention will develop and affect the heat transfer and other problems in a significant way. Among these are the nonequilibrium flow chemistry in both the shock layer and the boundary layer; the radiation from the high-temperature gas behind the shock which can be intensified by nonequilibrium effects; and the release of electrons in ionization which, because of their high degree of mobility, may tend to increase convective heating. At still higher entry speeds, the energy loss through gaseous radiation will begin to affect the flow field.<sup>3</sup> At all re-entry speeds, the electron density constitutes an important problem for communication and observation and deserves much attention in connection with nonequilibrium flow chemistry. The use of lift to sustain flight<sup>4</sup> and to increase the "entry corridor depth"<sup>1-3</sup> imparts renewed significance to the aerodynamic performance of spacecraft, particularly the lift/drag ratio at high altitude.

In order to bring out more clearly some of the major problems in hypersonics for re-entry, refer to Fig. 1, in which the velocity-altitude curves for a few typical re-entry trajectories are given. Also drawn in the same figure are boundaries

indicating or delimiting some of the important domains where the chemical or physical processes previously mentioned may arise. A look at the trajectories together with these boundaries may suggest how research in nonequilibrium chemistry and other flow processes will affect re-entry studies. Of course, boundaries of this type are quite arbitrary in nature, since they depend largely on the configuration and size of the body, the model atmosphere, and other factors. The present estimates are made for the stagnation region of a body with 1-ft nose radius, using a 1959 ARDC model atmosphere. The boundary A, which gives the maximum altitude for thermodynamic equilibrium in the shock layer, is based on the relaxation distance for the temperature to reach its equilibrium value behind a normal shock, similar to the estimates given previously by Feldman.<sup>6</sup> Curve B represents the upper altitude limit for an equilibrium boundary layer and is related to a recombination-rate parameter similar to that in Fay and Riddell's theory of the stagnation point boundary layer.<sup>7</sup> Except in the case of deep penetration typified by the ICBM trajectory, the critical portion of most re-entry flight paths (where maximum deceleration and heating occur) lies above both boundaries A and B. The nonequilibrium chemistry of the shock layer is particularly critical for lifting re-entry from superorbital speed, because chemical nonequilibrium tends to increase the temperature behind the shock and thus the radiative heat transfer, which is already substantial according to the equilibrium calculation. Referring to curve C, it is seen that the assumption of a continuum flow for predicting the critical motion and heating of re-entry vehicles may be largely valid. On the other hand, departure from the classical thin boundary-layer concept (curve D) as a result of external-vorticity and other rarefaction effects could be significant even at relatively low altitude. The boundaries E and F are related

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to the electronic-heat-conduction and radiation-loss effects, inferred, respectively, from the works of Adams<sup>8</sup> and Gouard.<sup>9</sup> These effects should be extremely important during atmospheric entry of large meteors.<sup>3</sup>

The main objective of this survey article is to provide a coherent account of the most recent work in hypersonic flows so as to reflect the current problem areas and the present state of the art. No attempt is made to review critically or appraise each piece of research, although, in fulfilling the present task, remarks and comments from the writer are inevitable and, in fact, necessary.

This survey will begin with investigations of ideal-gas inviscid aerodynamics and nonreacting boundary layers and then proceed to wake flows and rarefied gasdynamics. Studies involving nonequilibrium flows, dissociation and ionization in boundary layers, gaseous radiation, and other high-temperature aspects of flow problems will be discussed separately. The scope of this article is necessarily restricted by its title, and many fundamental contributions in fluid mechanics and physics which may relate to this field cannot be included. It may be noted that topics on plasma physics and magnetofluidmechanics are excluded, although many of the items discussed are related to these problems. An attempt has been made to limit the discussion mainly to full papers in journals, reprints, and reports that generally

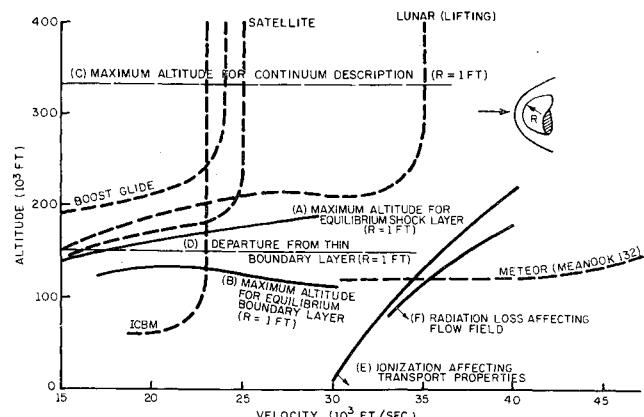


Fig. 1 Typical flight trajectories and boundaries delimiting some of the important domains

are available. A good deal of excellent work has been left out because of the limitation in space. The papers selected for the discussion may nevertheless be regarded as being representative of the state of the art.

## I. Inviscid Aerodynamics

During the crucial part of most of the entry trajectories, the ambient density generally is high enough to insure validity of the assumption of an inviscid outer flow. Under this high-density condition, the inviscid analyses will provide not only predictions for aerodynamic forces and moments but also the local flow conditions from which the skin friction and heat transfer rate can be determined. With the perfection of the characteristics method for three-dimensional flow,<sup>10</sup> the supersonic portion of the flow field can be determined for an arbitrary body, provided the solution for the nose region is known. Thus, the major problems that remain to be solved, from the viewpoint of numerical analysis, concern mainly the subsonic and transonic flow regions of blunt bodies and the supersonic conical flow field near the apex of pointed bodies. The following paragraphs deal with papers on inviscid flows of ideal gas with constant specific heats. Inviscid flows with chemical reactions will be discussed later under the heading of "Nonequilibrium Flow."

## Blunt Bodies

Detailed (numerical) analyses of supersonic flow field around blunt bodies have been carried out in the past mainly by the "inverse method,"<sup>11, 12</sup> in which a smooth shock shape is specified and the shape of the body is determined as part of the solution. This inverse approach becomes difficult for a problem in which the body of interest departs substantially from a simple shape such as an ellipsoid or hyperboloid. When an iteration procedure is employed to obtain solution for the "direct problem," the question of convergence of the procedure arises. As observed by Vaglio-Laurin,<sup>13</sup> this problem is related critically to the sonic-point singularity, which must, in fact, be answered in order to continue the solution obtained into the supersonic region downstream. An iteration scheme is proposed in which the sonic-point singularity is taken into account (by the *PLK* method) to insure convergence. When accurate knowledge of the flow field is not required, one can employ, of course, the comparatively simple integral method of Belotserkovskii. The method can be extended to treat flows around bodies with sonic corners as well as flow asymmetry.<sup>13, 14</sup> One may note that, in determining the solution by the integral method in the asymmetrical case, in which the stagnation point is not known a priori, Vaglio-Laurin<sup>13</sup> has had to make the assumption that the stagnation streamline comes through the strongest part of the shock. The loss of flow details by the integral method (as well as other methods) may be critical in certain applications. For example, the method does not allow for the existence of a second (recompression) shock wave that may occur downstream of a sonic corner. The question of how far one can count on the integral method has been examined recently by Traugott.<sup>15</sup> Among the many analyses on blunt-body flows without symmetry, a recent study via the inverse approach by Swigart<sup>16</sup> deserves attention. In this study, a paraboloidal shock with its axis inclined at a small angle to the freestream direction is assumed. Of course, the resulting flow field does not pertain strictly to the interesting case of flow around a symmetrical body at incidence. Nevertheless, the study uncovers a feature that may be essential for analyses of the asymmetrical problem: namely, the streamline wetting the body does not come through the normal shock, contrary to the tacit assumption made in Ref. 13. From the viewpoint of analysis, the semi-analytical method (based on the truncated series) used by Swigart which should be applicable also in a "direct problem" also is of interest.

## Pointed Bodies

In the category of pointed configurations, one should note the method of determining supersonic flow field around a cone of general section by Stocker and Mauger.<sup>17</sup> This numerical method, which is similar to the inverse method for the blunt bodies, has been applied with a considerable degree of success to the elliptic cones at zero yaw and circular cones at incidence at moderate Mach number. The method may become inadequate, however, as the surface is approached where a thin layer of high vorticity appears. The detailed structure of the vortical layer on circular cone at small yaw angle has been analyzed recently by Cheng<sup>18</sup> and independently by Wood.<sup>19</sup> The nature of the singularity and the manner in which it depends on the shock strength and gas property are worth noticing and may be helpful to further the numerical method.

## Low-Aspect-Ratio Wings

For low-aspect-ratio wings at high incidence, the crossflow concept is applicable. The problem has been considered by Cole and Brainerd, using the shock-layer theory.<sup>20</sup> Extension to the not-so-slender delta planform is made by

Messiter.<sup>21</sup> Analyses of this type perhaps may be used to assess the domain of applicability for the generalized equivalence principle of Sychev for slender bodies.<sup>22</sup>

#### Tip-Blunting Effect on Flows over Slender Bodies

From the viewpoint of convective heating, pointed noses and sharp leading edges are not desirable. The effect of nose blunting has been accounted for by the blast-wave theories, which are appropriate in the far downstream and for slender bodies.<sup>23, 24</sup> Recently, the complete flow field was treated analytically within the framework of the thin-shock-layer theory by Freeman for a sphere-cylinder.<sup>25</sup> The analysis provides a transition from the "free-layer" in the vicinity of the blunt nose to the "blast wave" far downstream. In many applications, the flow Mach numbers are not high enough to satisfy the assumption of an infinitely strong shock. Higher-order corrections for the finite shock strength therefore may increase greatly the range of applicability of the blast-wave theory, as is shown in a recent account by Lukasiewicz.<sup>26</sup>

Although the blast-wave concept is reasonably useful, its application may lead to embarrassing results for an "inverse problem" (in which the shock shape is specified). This has been demonstrated by Sychev<sup>27</sup> and Yakura<sup>28</sup> in their analyses of power-law shock waves. They both found that the slender body supporting a paraboloidal shock is quite different from the cylinder afterbody as supposed by the blast-wave theory. It will be of interest to investigate by the inverse method whether a slight alteration of shock shape will restore the body contour to something like a rod. The problem can, in fact, be carried out with the aid of the numerical characteristics method.

#### An Area Rule

An interesting hypersonic "area rule" for blunt-nosed slender bodies was given recently by Ladyzhenskii.<sup>29</sup> The rule is analogous to the now-classical transonic area rule: namely, bodies having equal area at each cross-section experience the same drag. The idea is taken over essentially from the entropy-layer concept of Chernyi<sup>23</sup> and others and is subject to a similar kind of limitations. The law follows readily from the observation that, on account of the extremely low density, the pressure remains uniform throughout the entire entropy layer at each section. Strictly speaking, the rule also implies no lift and no moment. This overly simplified picture serves to point out one of the serious aerodynamic control problems for slender bodies at hypersonic speed.<sup>3</sup>

## II Boundary Layers

#### Similitude

The aerodynamic and heat transfer problems of slender and thin bodies in high-altitude hypersonic flight involve the rather intricate interaction of three flow regions: the thick boundary layer, the low-density entropy layer, and the thin shock layer. The laws of similarity of the flows in question have been examined recently by Luniev.<sup>30</sup> The study differs from the work of Cheng and co-workers<sup>31</sup> in that the entropy layer associated with the bluntness effect is included in the viscous boundary layer; thus the assumption of the hypersonic equivalence principle which was made tacitly in the earlier work can be avoided. As noted also in the extensive study on the same subject by Guiraud,<sup>32</sup> this refinement necessarily introduces a dependence of the downstream flow field on the detailed flow structure of the nose region. Without further simplification, the resulting laws will, of course, leave very little freedom for actual application. The interesting viewpoints and details brought

out in these studies, however, should be of great value for further development of the subject.

#### Transverse-Curvature Effect

The analysis of displacement effect of the boundary layer on a slender body of revolution must take into account the transverse-curvature effect, when the boundary layer becomes comparable to the body in thickness. This is the subject of a recent study of Yasuhara,<sup>33</sup> in which a self-similar solution corresponding to the strong shock and boundary layer interaction is obtained. This solution is not applicable to a slender cone but to a slender power-law body that grows like  $x^{3/4}$ . The interaction problem for the slender cone also is treated in the same paper, using a local-similarity method. The behavior of the flowfield near the cone apex is of interest and requires further attention.

#### Local Similarity

Studies of the hypersonic boundary layer generally require solutions to the coupled nonlinear partial differential equations. The analyses may, however, be simplified by applying the local-similarity methods of Lees and others, which amounts to neglecting the effect of the tangential pressure gradients on the velocity profiles. A more recent application of the method was made by Kemp to obtain total convective heat flux in closed form.<sup>34</sup> An effective improvement of methods based on the local flat-plate similarity is given by Moore,<sup>35</sup> who also points out a connection between the local-similarity principle and the hypersonic shock-layer theory: namely, for slender bodies or nonslender but pointed bodies, an inviscid shock layer implies a boundary layer with local flat-plate similarity. The working of the local similarity in the hypersonic boundary layer may be regarded as a consequence of the persistence of the high-speed, high-density flow. This property, in the context of three-dimensional boundary layers, also implies the absence or smallness of secondary flow and is worth exploitation in conjunction with the vortical-layer effect on a yawed cone.

#### Cone and Cylinder Experiments

Experimental studies of recent years performed in high-speed wind tunnels and shock tunnels have provided much insight into hypersonic fluid dynamics, particularly on flows over flat plates and wedges. Recently, heat transfer to slender cones, including effects of yaw and nose-bluntness, was studied for airflow by Wittliff and Wilson<sup>36</sup> in a hypersonic shock tunnel at  $M \sim 12$ . Reasonable agreement is found with existing theories on yaw, displacement, and transverse-curvature effects (when these effects are small). The local flat-plate similarity appears to work best on the windward side of the cone at high incidence. Undershoot and overshoot in heat transfer rate are found along the slender afterbody cone, in qualitative agreement with the pressure distribution predicted by the theory of Chernyi and others. For the flow conditions and configurations studied, the vortical-layer effect does not appear to be significant. The combined bluntness and displacement effect on a yawed flat plate has been studied experimentally by Hall and Golian<sup>37</sup> and by Bogdonoff and Vas.<sup>38</sup> For sweepback angle less than  $45^\circ$ , the results in both pressure and heat transfer distributions appear to have borne out the "streamwise strip theory." Significant experimental results on yawed cylinders are obtained at a Mach number of about 4 and Reynolds number on the order of  $10^6$  (based on the cylinder diameter) by Beckwith and Gallagher.<sup>39</sup> The results show that yaw effects tend to bring about an earlier transition to the turbulent boundary layer, and, when transition occurs, heat transfer rate may increase with yaw angle.

### Transition from Laminar to Turbulent Flow

The boundary layer during certain critical parts of the re-entry trajectories presumably will become or begin to become turbulent. However, our understanding of the transition in a hypersonic boundary layer is far from being adequate. In this respect, the supersonic wind-tunnel study by Potter and Whitfield<sup>40</sup> may be cited. There, even small degrees of nose bluntness having negligible effect on measured pressure distributions are found to have a noticeable effect on boundary layer transition. The pronounced effects of boundary layer displacement and tip bluntness in hypersonic flow certainly are expected to be important factors affecting transition over slender bodies. From the viewpoint of the stability theory of the laminar boundary layer, the problem of analysis becomes more involved as the Mach number increases. Recently, Lees and Reshotko<sup>41</sup> have undertaken the stability analysis for a flat plate, including effects of fluctuations in transport properties and the viscous dissipation terms that are not important at low Mach number (considering only two-dimensional subsonic disturbances). Applications have been made only to an insulated flat plate at Mach numbers below 5.6. Another aspect of transition which recently has become of great importance is the transition to turbulent flow in the wake which will be discussed in the next section.

### III. Separated Flows and Wakes

Current interest in the subject of separated flow stems not only from the problems of aerodynamic control at high altitude but also from its bearing on the problem of the wake, which has become of recent interest as an observable. The earlier experimental and theoretical studies of Chapman, Kuehn, and Larson<sup>42</sup> on flow over steps and corners have brought much understanding to the subject. Recent developments in both theory and experiment pertain mostly to the substantiation or extension of the earlier concepts and findings.<sup>43-49</sup>

#### Separated Flows

On account of the rather complicated nature of the problem that involves interaction of the various flow regions, most analyses follow the Crocco and Lees and other similar integral approaches.<sup>43-46</sup> An analysis of this category is the recent treatment of flow separation from a surface by Erdos and Pallone.<sup>46</sup> One of the simplifying assumptions introduced in this analysis is the "free interaction," that is, the boundary layer separation is assumed to be a local phenomenon completely independent of the flowfield downstream. The pressure distribution over the separated-flow region determined in this manner therefore will be independent of the step size or deflection angle that induces the separation. Although this fact is commonly known,<sup>42</sup> the fact that such a seemingly oversimplified formulation does yield a meaningful solution is surprising. The unique result obtained also appears to offer an explanation of the existence of a minimum pressure jump to cause separation of supersonic flow from a smooth surface. This free-interaction concept also has been used to predict the wake angle of the laminar base-flow region for slender bodies.<sup>46</sup> In this case, the reattachment zone is assumed to be completely "free" from the upstream influence of the flowfield and body.

#### Base-Flow Region

An extension of Chapman's analysis of the laminar free shear layer is made by Denison and Baum<sup>47</sup> to include the initial boundary layer thickness at the separation point. The initial thickness effect is important for the study of free shear layers separated from slender bodies. With the aid of Chapman's dividing-streamline concept, the solution

is applied to determine the length of the base-flow region. The results obtained by this analysis for the wake angle and base pressure are independent of the Reynolds number. Clearly, the analysis is applicable only at high Reynolds number before transition occurs. By contrast, the corresponding results obtained by Erdos and Pallone based on the free-interaction model that emphasizes the displacement effect show a definite Reynolds number dependence.

It should be pointed out that an understanding of the base-flow problem is essential, inasmuch as the steady or unsteady state of the base-flow region will affect significantly the fluid mechanics and thermochemistry of the wake. In this respect, the question on the temperature inside the recirculatory-flow region is of particular importance. This is not only because the temperature will control the length of the closed wake<sup>42, 47</sup> but also because, when this temperature is high, as a careful study will show, the recirculatory-flow region may serve as an ideal site for electron production.

#### Laminar and Turbulent Wakes

Turning now to the wake problem itself, both purely laminar<sup>6, 60</sup> and fully turbulent wakes have been treated.<sup>51</sup> Since chemical kinetics were not considered, the main problem was reduced essentially to one of cooling of a nonuniformly heated core by laminar or turbulent transport process. In Lees and Hromas' analysis,<sup>51</sup> the eddy diffusivity in the turbulent wake is assumed to relate to the local momentum defect through an extension of Townsend's model for a low-speed isothermal wake. Their result agrees well with the wake-width measurement of Slattery and Clay,<sup>52</sup> in spite of the many approximations and idealizations introduced in the analysis. The effects of unsteadiness, three-dimensionality, the Lewis and Prandtl numbers, as well as other effects, have been treated by Bloom<sup>53</sup> and Steiger.<sup>54</sup>

#### An Equivalence Principle

An equivalence principle for the hypersonic wake, pointed out recently by Sychev,<sup>55</sup> applies to the flowfield far downstream of a body, including the viscous (laminar) wake region as well as the oblique shock transition zone. Accordingly, bodies having the same drag have identical wake structure, irrespective of speeds and nose drag coefficients. The principle's range of validity and the questions as to its applicability to turbulent wake and wakes with chemical reactions are worth exploring.

#### Transition to Turbulent Wake

One of the problems of the hypersonic wakes which requires more critical study is how and when transition to a turbulent wake takes place. The schlieren motion-picture records of Slattery and Clay<sup>52</sup> of the hypervelocity trails of a projectile in air, as well as radar echo returns from satellite re-entry obtained by Lin,<sup>56</sup> indicate that a transition zone can be identified at some distance downstream of the body. By examining the radiation intensity from self-luminous wakes, Hidalgo and Taylor<sup>57</sup> report a transition Reynolds number based on local flow conditions and the length of the "laminar run" on the order of  $10^4$ . On the other hand, Goldberg and Fay<sup>58</sup> have compared the photographs of the self-luminosity trails of hypervelocity pellets in air (using a "race track" technique) with those obtained at low speed in liquid. Their comparison suggests that shedding of vortices, such as vortex loops, from the base region may be the source and cause of turbulence in the subsonic as well as the hypersonic wakes. The inference would imply that the answer to the wake-transition problem is to be found in the steady or unsteady nature of the base flow, and the possibility of transition after a laminar run would have to be ruled out.

#### IV. Low-Density Fluid Dynamics

For the study of heating and decelerations of re-entry vehicles, a continuum description of the flow field may seem to be sufficient, but the departure from the thin boundary layer concept may have to be accounted for at high altitudes. However, the design and use of artificial satellites depend to a great extent on our knowledge of the free-molecule and transition-flow regimes pertaining to the rarefied environment. Research in the past few years has contributed much to the understanding of these regimes.<sup>59, 60</sup> Certain important aspects of the problems related to the free-molecule end have been reviewed recently by Patterson,<sup>61</sup> and the present state of the art in the transition regime has been discussed within the context of shock structure in a review by Talbot.<sup>62</sup> The Third International Symposium on Rarefied Gas Dynamics, which was held in Paris in June 1962, in keeping with the spirit of the previous meetings provided an ample forum for the presentation of up-to-date results and studies. Many of these recent works, of which the writer is unable to give account here, will be found in the forthcoming proceedings of the symposium.

##### Free-Molecule Flow and Gas-Surface Interaction

Because of the lack of basic knowledge of the interaction between gas particles and surface, the current theory of free-molecule flows may in certain instances become superficial. One may note in particular that the average atmospheric density derived from satellite deceleration data is not very precise because, in part, of the uncertainty in the surface accommodation coefficients. The classical experimental and theoretical methods for determining thermal accommodation coefficients have been reviewed by Hartnett<sup>63</sup> and more recently by Wachman.<sup>64</sup> The range of the particle energy attainable in the classical experiments is, however, too low to be applicable to the satellite problem, and such basic information must be obtained from experiments with gas particles having velocity matching the satellite speed. Neutral molecule beams having particle kinetic energy of the order of 1000 ev or higher can, in fact, be generated in the laboratory;<sup>65</sup> these are suitable, however, only for studying the high-temperature transport properties. The future study will have to depend on techniques of developing a molecule beam in the desirable particle-speed range (1 to 10 ev) which are still in their infancy at the present time.<sup>66, 67</sup>

##### Application in the Ionosphere

At an altitude of 1500 km, where the degree of ionization can be as much as 50%, application of the free-molecule analyses must take into account the interactions of the ionized particles with the bodies.<sup>61, 68, 69</sup> The charges acquired by the satellites can result in drag forces due to interaction with the ions as well as to the unsymmetrical currents through the satellite in the presence of the earth's magnetic field. A better understanding of the flow field will permit a better evaluation of the ionosphere measurement made aboard such vehicles.

##### Flow and Heat Transfer between Parallel Plates at Low Density

Extensive studies of the transition regime in recent years have indicated that treatments based on formal expansion schemes (such as the Burnett and 13-moment expansions) do not yield improvement to the Navier-Stokes theory. Neither has the first-order collision theory actually established its validity as a first-order correction to the free-molecule flow.<sup>70</sup> It is clear that studies involving the transition regime must deal one way or another with the Boltzmann equation governing the "distribution function."

A number of approximate methods have been introduced in the past few years to overcome the difficulty of solving the Boltzmann equation, as well as to simplify the equation itself. The simple problem of flows between parallel plates with relative motion (corresponding to the classical Couette flow) provides an ideal testing ground for these methods. The various methods were compared recently by Willis under the framework of this problem,<sup>71</sup> which is further simplified by the restriction to slow plate motion and equal plate temperature as well as by adoption of the Krook kinetic model. In the Krook model, the collision integral in the Boltzmann equation is simplified, and the equation is reduced to a form of the relaxation equation. For this linearized Krook model, an exact numerical solution can be obtained which then is used as a basis for the comparison. Although the moment methods of Gross and co-workers<sup>72</sup> and of Lees<sup>73</sup> provide reasonable approximation, substantial improvement in their accuracies can be obtained by iteration based on an integral-equation formulation.<sup>71</sup> The need for improving Lees' method is not surprising, since the structure of the "Knudsen layer" is completely lost in Lees' moment method. Observing this, Shen<sup>74</sup> has developed recently a moment method in which the distribution function has a built-in property that allows for a "nonanalytic behavior" related to the Knudsen layer and also guarantees the correct asymptotic limits at high and low Knudsen numbers.

The problem of heat transfer between two stationary plates without mean-flow velocity is considerably simpler. In fact, for the "Krook-Boltzmann equation," an exact solution has been obtained by Willis.<sup>75</sup>

##### Shock Structure

The problem of the shock wave structure provides one of the simplest situations in which mean flow properties vary considerably over a distance of the mean-free path. Naturally, its analysis will require the use of the Boltzmann equation or something equivalent. A number of approximate methods have been applied to treat the shock transition zone and are discussed in the review by Talbot previously mentioned.<sup>62</sup> One should observe, however, that no solution to this problem yet has been obtained (even for the crudest kinetic model) which is accurate enough to provide a standard for comparing the various methods, nor have experimental studies yielded data that are adequate for assessing the methods. In fact, the sole experimental information pertaining to strong normal shock consists of only one value of shock thickness, which is deduced from an optical reflectivity measurement at  $M = 5$  by Hansen, Hornig, and co-workers.<sup>76</sup> Although this value appears to come close to those predicted by the moment method based on the bimodal distribution (Mott-Smith), there are several assumptions implicit in the interpretation of the experimental datum which may not be valid at high shock strength.

An interesting approach that avoids the formalism of the Boltzmann equation is the Monte Carlo method employed by Haviland.<sup>77</sup> The basic idea explored is to follow a large number of molecules through random collisions and then to determine the distribution function of these molecules.

Very encouraging results were obtained recently by Chahine<sup>78</sup> and Liepmann and co-workers<sup>79</sup> in the analysis of the shock-transition zone using Krook's collision model. The same model was employed to study the shock structure by Koga<sup>80</sup> in the previous year. The analyses of Refs. 78 and 79 differ from Koga's essentially in that the reciprocal of the relaxation-time scale appearing in the Krook equation, which may be interpreted as a mean collision frequency and was taken as a constant in Koga's work, is allowed to vary from point to point. The results of Liepmann and co-workers for the velocity profile (first iteration) show little difference from the Navier-Stokes solutions on the higher-density side of the shock, even for Mach number as high as 10. Using

the Navier-Stokes equations, analytic solutions of the shock-wave profile were obtained and used recently by Sychev<sup>55</sup> and Bush<sup>51</sup> to study the viscosity-law and Mach-number dependence.

### Leading-Edge Slip Effect

The flow over a semi-infinite flat plate constitutes a basic problem for aerodynamic and heat transfer studies. The limiting behavior of the surface pressure and heat transfer rate near the leading edge of a flat plate in rarefied gas flow is described by the first-order collision theory.<sup>70</sup> As an attempt to provide some knowledge for the transition region downstream, Oguchi<sup>82</sup> has employed previously a hypersonic viscous-layer model consisting essentially of a viscous boundary layer bounded by the flat plate and the shock. A rather surprising result of the analysis based on this model was that the shock angle, and consequently the pressure, starts with a finite value at the leading edge. More recently, Oguchi<sup>83</sup> has included in his analysis the wall slip and temperature jump effects and has obtained corrections to the surface pressure and other quantities which are in a direction consistent with the results of the first-order collision theory.<sup>70</sup> However, when the wall slips are important, one must consider also other slip effects at the shock which result from the modification of the Rankine-Hugoniot relations by the shear-stress and heat-conduction effects immediately behind the shock.<sup>84</sup> These slip effects at the wall and at the shock, as well as the thickness (and curvature) effects of the shock itself, are analyzed in a recent study by Pan and Probstein.<sup>85</sup> The wall slip also has been studied by Street in the problem of shock boundary layer interaction.<sup>86</sup> Using low-density hypersonic shock tunnels, Nagamatsu and co-workers<sup>87</sup> and Vidal<sup>88</sup> have studied the leading-edge slip problem with heat transfer data. Vidal succeeded in correlating his data for different pressure levels into a single curve that includes the shock and boundary layer interaction value at one end.

### Higher-Order Boundary Layer Theory

In the absence of adequate treatments based on the kinetic theory, it is reasonable to use the Navier-Stokes equations to penetrate as far as is permissible into the transition regime from the continuum end. Apart from the flat-plate leading-edge problem discussed in the preceding paragraph, most studies using the continuum approach concern the blunt-body problem, specifically, the stagnation region. Existing work on the blunt-body problem falls into two main categories. In the first, the classical boundary layer theory is modified to account for the outer-flow vorticity and other higher-order effects. In the second category, which will be discussed later, the full Navier-Stokes or the simplified equations are integrated numerically along the axis of symmetry on the basis of an assumption of local similarity.

A critique of existing theories on the subject has been given recently by Van Dyke<sup>89</sup> with emphasis on work belonging to the first category. A consistent analysis of the departure from the thin-boundary-layer theory must include the effect of external vorticity, the displacement effect on the outer-flow velocity, the displacement effect on the pressure, the finite-curvature effect, and the effects of slip and temperature jump. The last three effects are relatively small and, in fact, can be neglected under the thin-shock-layer approximation and the assumption of a highly cooled surface,<sup>84</sup> at least for a blunt body. Discrepancies, however, are found among various theories regarding the slip effects. Although Rott and Lenard,<sup>90</sup> as well as Van Dyke,<sup>89</sup> advocate, with support of their own results, that slip effects are not entirely negligible, Maslen's analysis<sup>91</sup> reveals definitely a negligible slip effect.

### Viscous Blunt-Body Problem

The second category includes analyses pertaining to the viscous layer and the incipient-merged layer, as well as the merged-layer models. Among recent works belonging to this class, the study of mass transfer by Goldberg and Scala<sup>93</sup> and the analysis of Levinsky and Yoshihara<sup>94</sup> may be cited. The latter analysis provides a detailed description of the shock-transition zone as a part of the solution. The method of Ferri and co-workers<sup>92</sup> involves a procedure of matching the inner and outer solutions but does not specify that the boundary layer be thin. Its working range presumably is comparable to the viscous-layer theory.

It must be pointed out that the local-similarity assumption that underlies most analyses of this category can be justified only in the limit of a vanishingly thin shock layer. Thus, the analyses are no more accurate than the approximation employing the thin-shock-layer concept. This is the reviewer's argument in support of Shidlovsky,<sup>95</sup> who recently treated the stagnation region by the shock-layer approximation. By a systematic expansion in terms of  $(\gamma - 1)/(\gamma + 1)$ , an approximate, analytic solution is obtained which includes the viscous and heat-conduction effect immediately behind the shock. This result should be identifiable with that arrived at earlier by Cheng.<sup>84</sup> However, Shidlovsky's formal expansion does not work in the higher Reynolds number regime, as it precludes the boundary layer phenomena near the surface. To the higher Reynolds number regime, a substantially different method of analysis has to be applied.<sup>84</sup> Recently, the shock-layer approach was used by Cheng and Chang to study flow around yawed cylinders.<sup>96</sup> One desirable feature of the thin-shock-layer approach may be noted: it can be adopted readily to analyze problems beyond the stagnation region because the treatment is based on an equation of parabolic type.

### Stagnation Point Heat Transfer at Low Density

Measurements of stagnation point heat transfer have been made in high-speed wind tunnels and shock tunnels to spheres<sup>92, 97, 98</sup> as well as to yawed and unyawed cylinders.<sup>88, 97</sup> Although certain discrepancies do exist among these data, as well as among analyses, the general trend of the data does bear out the theoretical prediction. A disagreement remains, however, with data in the higher Reynolds number range (where the departure from the boundary layer prediction actually is quite small). In this range, the data of Ferri et al.,<sup>99</sup> along with their own prediction, appear to be considerably higher than those of the others.<sup>84, 89, 97</sup> One may note that, for the temperature range encountered in all of these experiments, appreciable vibrational excitation of the molecules is to be anticipated. In fact, Ferri<sup>99</sup> reported that data correlation is rather sensitive to the value of the specific-heat ratio chosen. The current dispute on the (small) vorticity-interaction effect could very well suggest the importance of studying the aspects of vibrational (and perhaps also rotational) relaxation. This point has been amplified in a recent study of shock-wave structure by Scala and Talbot.<sup>100</sup>

### V. Nonequilibrium Flows (Inviscid)

The importance of nonequilibrium flow research, as suggested in Fig. 1, is quite evident. Studies assuming a single dissociation-recombination reaction have unquestioned value in various respects. However, this simplified picture of chemical kinetics is inadequate for high-enthalpy air flows of current interest. Although our knowledge concerning rates of many important reactions has yet to improve, the information at hand, reviewed by Wray,<sup>101</sup> does warrant more realistic analyses that take into account simultaneously the various reaction processes. The coupling among the

various reactions, as well as the gasdynamics of the flow system, indeed is significant, and its importance has been amplified duly by many of the recent results.<sup>102-106</sup>

### Analyses of Flow with Coupled Reactions

The gasdynamics of flows in nozzles and about bodies are affected strongly by the nonequilibrium processes. On account of the nonlinearity and complexity in the analyses, a numerical approach to the exact solution of the problem is inevitable. Numerical solutions for the relaxation zone behind normal shocks have been obtained by a number of investigators in previous years. The more difficult problem of nozzle flows has been studied previously by many workers, treating mainly dissociating diatomic gases. The method for analyzing nozzle-expansion flows has been extended recently by Eschenroeder and co-workers<sup>102, 113, 114</sup> and others<sup>103</sup> to analyze coupled chemistry of air, including ionization. For the study of the nonequilibrium flow field around a smooth blunt body, the inverse method of Garabedian and Van Dyke was applied previously by Lick, considering a single dissociation-recombination reaction. The method was generalized recently by Marrone<sup>104, 106</sup> to permit analyses of nonequilibrium air flow with coupled chemical reactions. These exact numerical solutions not only provide a sound basis for the study of nonequilibrium flows in nozzle and around bodies but also serve as touchstones for the streamtube and other simple methods.

The studies of Hall and co-workers<sup>102, 104</sup> and others<sup>103, 105</sup> based on these numerical analyses have brought out clearly that the bimolecular-exchange (or shuffle) reactions involving the production and consumption of nitric oxide (NO) are important for the chemical kinetics of N as well as of NO. In air flow behind the bow shock of a blunt body, the exchange reactions, on account of their comparatively low activation energies, provide an effective path for the production of N after oxygen dissociation begins. During flow expansion around bodies, as well as in nozzles, on account of the low density, atom concentrations tend to freeze at levels well above the local equilibrium values; however, the bimolecular-exchange path for N-atom removal can, in this instance, effect a delay in nitrogen freezing.

### Binary Scaling Law

For the simulation of the high-enthalpy nonequilibrium air flows with coupled chemical reactions, a scaling law is highly desirable. If all chemical reactions occur through binary (two-body) collisions, the whole flowfield, including the nonequilibrium chemistry, corresponding to a fixed freestream velocity, can be scaled by keeping fixed the product of body scale times freestream density, as in the Reynolds number scaling. Gibson observed that, in most cases where nonequilibrium effects are important, three-body processes such as oxygen and nitrogen atom recombinations are not contributive.<sup>107</sup> Then binary scaling will apply. Of course, validity of this law requires a relatively low freestream density. Studies of inviscid flows around blunt bodies of reasonable sizes indicate that scaling is possible over a wide range of altitudes.<sup>104, 108</sup> Binary scaling also should be applicable to slender bodies, with or without tip bluntness. Application of the binary-scaling law in conjunction with the hypersonic similitude for slender (pointed) bodies is worth exploring. The hypersonic similitude would allow changing the freestream velocity with a corresponding change of thickness ratio.

### Mapping of Normal-Shock Solutions

When binary scaling applies, the analysis of nonequilibrium blunt-body flows also can be simplified greatly. Gibson and Marrone<sup>108, 109</sup> note that the static enthalpy varies little

both along a streamline of blunt-body flows and in the relaxation zone behind a normal shock. Under the shock-layer approximation, a mapping of the normal shock solution can, in fact, be performed so that the flow problem involving coupled chemistry reduces to an analysis of shock relaxation zones. The mapping is not applicable near the blunt-body surface where the enthalpy drops appreciably.

### Rapid Freezing

When recombination is controlled by a relatively slow mechanism such as three-body collisions, the nonequilibrium process during flow expansion begins and ends rapidly enough that a sudden freezing approximation can be applied to determine the frozen chemistry. The approximation (which was proposed by Bray and independently by Hall and Russo) consists of joining a frozen and an equilibrium flow region at a certain freezing point. Rapid freezing of the vibrational degree of freedom also will occur in nozzle flows, leaving the vibrational temperature many times higher than the translational temperature, as revealed by the recent analyses of Stollery and Smith.<sup>110</sup> The freezing as well as other aspects of nozzle flows were reviewed recently by Bray.<sup>111</sup>

The freezing during expansion around blunt bodies is of great importance in the study of flow chemistry over afterbodies and in wakes. One must note, however, that the sudden freezing generally is not a good approximation to the flow expansion around a blunt body, since the flow chemistry along a streamline behind a bow shock is not dominated by the recombination process alone as is the nozzle flow, and the freezing in a dissociation-controlled flow is relatively gradual.<sup>104, 108</sup> On account of the large amount of energy which is locked up with the frozen degrees of freedom, one may anticipate a substantial change in the hypersonic flow field downstream of a blunt nose as a result of chemical non-equilibrium. An idealized model of this sort, assuming full equilibrium in the nose region, has been considered by Whalen.<sup>112</sup>

### Electron-Ion Recombination

The processes of electron-ion recombination in high-temperature air for nozzle flows as well as flow along streamlines near blunt bodies have been studied recently by a number of workers.<sup>105, 113, 114</sup> The flow of highly ionized argon through a nozzle has been investigated by Bray.<sup>115</sup> In most examples, departures from equilibrium are gradual, and sudden freezing does not occur. This is because the dominant two-body mechanism ( $\text{NO}^+ + e^- \rightarrow \text{N} + \text{O}$ ) is relatively fast and has a large inverse temperature dependence. In fact, the electron concentration determined by the latter process over the afterbody is self-limiting, as pointed out previously by Lin.<sup>116</sup> An exception to this trend is the result obtained for the flow around a blunt body for a lunar probe re-entry condition,<sup>114</sup> in which case the electron concentration over the afterbody is frozen practically at the high stagnation value.

### Kinetics of Coupled Vibration and Dissociation

One major difficulty underlying the current studies of nonequilibrium flow chemistry for re-entry conditions is concerned with the uncertainty that arises from the extrapolation of the experimentally determined rate constants to the extreme temperatures and low densities. Experimental data on dissociation, for example, usually are obtained under conditions where considerable vibrational excitation has occurred. The proper interpretation of the rate data from experiment and the appropriate law for extrapolation to the higher temperature range of interest require a knowledge based on a chemical kinetics model that takes into

account the coupling between vibration and dissociation. Hammerling and co-workers,<sup>117</sup> in their work on radiation behind a shock front, observe that the degree of vibrational excitation has an effect on the rate of dissociation, since dissociation from a higher vibrational level requires less energy and thus will proceed faster. On the other hand, the rate of vibrational excitation will be determined in part by the rate of dissociation. This is because dissociation takes away the more energetic vibrators, thus reducing the average vibrational energy of the remaining molecules in a manner analogous to evaporation cooling. The recent investigations of Treanor and Marrone have taken both of these aspects into account.<sup>118, 119</sup> However, implicit in the existing analyses are two critical assumptions. One is the use of the Boltzmann distribution in the vibrational mode; the other is the specification of the relative dissociation cross section for the various vibrational levels. The removal of these two assumptions is a formidable task and has received some preliminary attention from various investigators, as described in Refs. 120-123. Presumably, a similar coupling exists between the electronic excitation and ionization.

## VI. Mass and Heat Transfer with Dissociation and Ionization

Control of the heating, communication, and other crucial problems of re-entry calls for an understanding of the chemical and transport processes in boundary layers and in wakes. Recent work in this area has enlarged significantly the scope of the previous studies of dissociation and ablation in boundary layers and has focussed attention on the ionization effect on heat transfer. In the preceding section, nonequilibrium inviscid flows were dealt with. In the following section, nonequilibrium chemistry in the boundary layer will be considered.

### Nonequilibrium Coupled Reactions in Stagnation Point Boundary Layer

Inasmuch as nonequilibrium coupled reactions in air have been studied for inviscid outer flows, it is of interest to examine the important question of whether coupling of the various reactions is important for the boundary layer. The only study of this sort, to the writer's knowledge, is the recent work of Moore and Pallone for the stagnation point.<sup>124</sup> In this work, air is treated as a mixture of O, O<sub>2</sub>, N, N<sub>2</sub>, and NO, assuming vibrational equilibrium. The work also includes a rather extensive calculation of transport properties (by W. L. Bade). Numerical results are obtained for a noncatalytic surface of 1-ft nose radius at 200,000-ft altitude and  $M = 25$  over a wide range of wall temperature. Strictly speaking, their analysis does not include the exchange (shuffle) reactions that are significant in inviscid nozzle and blunt-body flows.<sup>102-105</sup> However, the effects of these reactions are found to be small in this case by examining their contribution with the aid of rate equations and the solution that has been obtained. The ineffectiveness of the exchange reactions apparently is related to the fact that the density in the stagnation point boundary layer, instead of decreasing as during flow expansions, increases rapidly toward the cold surface, favoring three-body recombination processes. The exchange reactions may, however, become more important in a boundary layer where expansion occurs in the outer flow.

From the foregoing, it appears that the air chemistry can be simplified considerably in the stagnation point boundary layer, but that the coupling among the remaining reactions is not unimportant. These reactions consist of the dissociation and recombination of oxygen and nitrogen as well as the formation of NO from N and O. In this connection,

one may note that the usual idealization of the air chemistry by a binary mixture of "air atoms" and "air molecules" may lead to significant error. As observed by Fenster and Heyman,<sup>125</sup> this ambiguity appears most critically in the definition of the heat of formation for the "air atoms." Some improvement may be gained by using a heat of formation which allows for species variation across the boundary layer. This modification, which may be quite significant for a non-equilibrium boundary layer, has not been explored fully.

According to Moore and Pallone's results, the nonequilibrium heat transfer rate for the noncatalytic surface amounts to only 30% of the value based on equilibrium chemistry (and 140% of the value based on frozen chemistry). This indicates the importance of surface catalycity in the speed and altitude range considered.

### Surface Catalycity

Some valuable information on catalytic properties of solid surfaces in promoting atom recombination has been provided by a number of experimental studies in recent years.<sup>126-128</sup> In this instance, the determination of the recombination coefficients for atomic oxygen on silica and on platinum which was made at surface temperatures as high as 1120°K may be cited, respectively, by Greaves and Linnett<sup>126</sup> and Hacker and co-workers.<sup>127</sup> There are many gaps to be filled in the actual determination of recombination coefficients, however, inasmuch as the chemical kinetics on the surface depend critically on the local temperature, pressure, the nature of the surface, and the species considered. There are, in addition, many other factors making the interpretation of the experimental data difficult. Among these are the strong effect of the "impurity" of the surface, the coupling of the surface recombinations of nitrogen and oxygen, and the possible variation of the heat of formation due to the vibrational or electronic excitation of the adsorbed particles, by analogy with the gas-phase reaction.<sup>129</sup> A more recent experiment, in fact, indicates that catalycity is affected strongly by the pre-test history of the surface.<sup>130</sup>

Heat transfer measurement offers an attractive way to study wall catalycity. To do so, one must rely on an appropriate fluid-dynamic theory that relates heat transfer data to recombination coefficients. Thus Hartunian and Liu<sup>131</sup> analyze the concentration field around a cylinder with surface recombination at slow motion (corresponding to the classical Oseen approximation), and Chung and co-workers<sup>132</sup> obtain a solution to the frozen boundary layer in a dissociated freestream over a flat plate that has a discontinuity in surface catalytic efficiency. The latter solution should be useful for interpreting experiments on the relative catalytic efficiencies of different surfaces.

The possibility of using catalytic surfaces to determine atom concentration in a flow has been discussed by Rosner<sup>129</sup> and Hartunian.<sup>133</sup> In spite of the lack of general knowledge of the surface interaction, the method has its basis in the fact that the surface recombination (in a steady state) is controlled to a large extent by both the surface chemistry and the local diffusion process. At high enough Reynolds number, one may have a fully catalytic-wall effect irrespective of the recombination coefficient, provided the coefficient itself is not extremely low, of course. In addition, the difference in catalytic efficiencies between two different surfaces also may be used to eliminate the uncertainty in their absolute values. In order to provide a direct connection between surface recombination and the freestream atom concentration, the density level should be low enough to maintain a frozen gas-phase chemistry throughout the entire flow field but high enough to avoid the uncertainty brought about by the thermal accommodation coefficients in the free-molecule and the transitional flow regimes.<sup>133</sup> The range of ambient density or Reynolds number in which such a probe is applicable therefore is not very wide.

### Nonequilibrium Flow Chemistry in the Rarefied Gas Regime

Tacitly assumed in almost all existing analyses of non-equilibrium boundary layer at the stagnation point is the equilibrium condition at the "outer edge." When the gas-phase chemistry becomes frozen as the degree of gas rarefaction increases, not only does the boundary layer theory have to be modified to take the vorticity-interaction effect into account as discussed previously, but also the species concentration at the "outer edge" may have to be reduced and modified because of the nonequilibrium dissociation in the outer flow. As a result, the use of noncatalytic surfaces will not be as effective in reducing the heat transfer as predicted by the boundary layer theory. In fact, according to Chung's analysis<sup>134</sup> based on the viscous-layer model, which was carried out for a 1-ft sphere and for a binary mixture of "air molecules" and "air atoms" at a near satellite speed, the reduction in convective heating by the noncatalytic surface is almost negligible at an altitude of 280,000 ft. Chung's result also shows that there is a substantial decrease in the general level of dissociation as a result of lowering the flowfield temperature due to heat conduction to the relatively cold surface. Therefore, one may anticipate in the hypersonic rarefied-gas regime a situation in which the gas-phase reaction is frozen, and this is indeed favorable for using the catalytic surface as a concentration probe, as previously discussed.<sup>133</sup>

When the viscous and heat-conduction effects are felt through the major portion of the shock layer, the ordinary Rankine-Hugoniot shock relations also must be modified consistently to include these effects, as discussed previously.<sup>59, 84</sup> In this regard, one may anticipate a concentration jump across the shock, resulting from the upstream diffusion of the atom species that are produced inside of the shock layer. Using a viscous-layer model, Goldberg and Scala<sup>93</sup> study mass transfer in the shock layer. The conclusions drawn there must be taken with caution not only because the viscous-layer model used does not account for the concentration jump and other similar effects across the shock, but also because an equilibrium chemistry was assumed, which is certainly far from reality in the light of Chung's study, as well as the studies of nonequilibrium chemistry for inviscid air flow discussed before.<sup>104-106</sup>

### Nonequilibrium Boundary Layer over a Flat Plate

Most existing analyses on boundary layers with nonequilibrium chemistry concern the stagnation point. From the viewpoint of lifting re-entry, however, the nonequilibrium boundary layer such as that over a wing at high incidence deserves attention. A characteristic feature of the hypersonic boundary layer in regions other than the stagnation region is the overshoot in the temperature profile resulting from viscous dissipation. Since the rates of dissociation in most cases depend rather critically on the local temperature (in an Arrhenius way), dissociation will take place most vigorously where the temperature attains its maximum. This behavior is most prominently displayed in the hypersonic boundary layer over a flat plate. In a recent study of this problem for the Lighthill-gas model, Rae<sup>135</sup> considers small departures from the frozen chemical state of the leading-edge region, which permits him to deal fully with the rate equation. Rae obtains a simple approximate solution based on the delta-function-like behavior of the dissociation rate. The analysis suggests that an equivalent reaction surface could be introduced for handling chemical reactions in hypersonic boundary layers over slender bodies, especially when the gas-phase recombination is not important.

### Nonequilibrium Chemistry in Viscous Wakes

The analysis of a fully developed turbulent wake in chemical equilibrium by Lees and Hromas<sup>51</sup> is useful in that it provides

conservative estimates for engineering applications. A more critical study of the electron distribution in the wake will come from analyses of the nonequilibrium chemistry. Recently, Bloom has studied the nonequilibrium ionization in a laminar wake.<sup>136</sup> To simplify the analysis, the governing differential equations are satisfied only along the axis. The results exhibit certain interesting features, including the electron and temperature overshoots. The latter is found to result from nitrogen recombination. In this study, Bloom has assumed a Lewis number of 1.4; the coefficient for the electron and/or ion diffusion is not discussed, however.

### Effect of Ionization on Stagnation Point Heat Transfer

Because of the presence of electrons and ions, there are strong influences on the energy and mass transport even at a considerably low level of ionization. As the temperature exceeds 8000°K with only a small degree of ionization, the thermal conductivity increases rapidly and becomes proportional to  $T^{5/2}$  as for the case of a fully ionized gas. In addition, energy also is transported by diffusion of charges in the form of ionization energy, which is determined essentially by the interaction of the charged and neutral particles. Because of the strong mutual attraction, the electrons and ions tend to diffuse in pairs (ambipolar diffusion). This results in doubling the rate of diffusion for the ions. In the earlier work, Adams<sup>8</sup> has made a simplified study of these effects on the chemically frozen boundary layer at the stagnation point with a fully catalytic surface, considering a mixture of three components (atoms, molecules, and ions). One important fact demonstrated by Adams' analysis is that, unless velocities in excess of that for lunar probe re-entry are considered, the increase in convective heating due to ionization is not very large, since the temperature in the boundary layer on a relatively cold surface is considerably lower than the stagnation temperature in the outer flow. Also noted in Adams' study is the insensitivity of the ionization effect on convective heat transfer to the change in altitude.

The stagnation point heat transfer also has been calculated by Hoshizaki<sup>137</sup> and Cohen<sup>138</sup> for speeds up to 50,000 fps. Both authors assume chemical equilibrium and make use of the equilibrium transport properties of air given by Hansen.<sup>139</sup> Using the more up-to-date information on transport properties of equilibrium air calculated by Yos,<sup>140</sup> Pallone and Tassell<sup>141</sup> recalculate the stagnation point heat transfer. All results show little difference from that extrapolated from Fay and Riddell for equilibrium dissociated (un-ionized) air.

On the other hand, Scala<sup>142, 143</sup> calculates the convective heat transfer over the same speed range, assuming equilibrium chemistry and considering mainly the dissociation and ionization of pure nitrogen. Scala's calculation yields an increase of heat transfer due to ionization which is considerably larger than those obtained by the others. As is apparent from a recent review by Fay<sup>144</sup> and the more recent analysis by Pallone and Tassell,<sup>145</sup> this large difference in the heat transfer prediction results not from the model of chemistry and methods of analyses but primarily from the laws and estimates assumed for the transport properties. More specifically, the difference lies in the models used for the charge-neutral collisions that control the diffusion of ions and thus the ionization energy.

In Scala's work, an induced-dipole (polarization) force is assumed in the determination of the N-N<sup>+</sup> collision cross section. As pointed out by Fay, the assumption may not be appropriate for the present application where polarization energy is small in comparison with  $kT$ . Instead, Fay proposes the charge exchange model<sup>146</sup> to compute diffusion coefficients for the charge-neutral collisions at high temperature. Assuming that N-N<sup>+</sup> and N<sub>2</sub>-N<sub>2</sub><sup>+</sup> have about the same charge-exchange cross section, Fay<sup>144</sup> and Yos<sup>140</sup> use

cross sections that are one or two orders of magnitude larger than those obtained from the extrapolation based on the induced-dipole model (as well as that for the neutral-neutral collision). Thus the charge-exchange collision leads to a much weaker rate of diffusion of the ion-electron pairs. Therefore, assuming the charge-exchange model, the increase of thermal conductivity due to electrons will be offset partially by the decrease in diffusivity of the ions. Thus, these compensating effects make the ionization even more insignificant for convective heat transfer study than indicated by Fig. 1, whereas, in Scala's model, ion diffusion will appear to augment energy transport to the body. It may be noted that, in Hansen's calculation<sup>139</sup> underlying the predictions of Hoshizaki<sup>137</sup> and Cohen,<sup>138</sup> the cross sections assumed for the charge-neutral and Coulomb interactions are quite different from those of Fay<sup>144</sup> and Yos.<sup>140</sup> Apparently, the discrepancies are mutually compensating.

As pointed out by Fay, the detailed studies based on equilibrium chemistry<sup>137-141, 145</sup> may not be relevant to the flight problem (see also Fig. 1). One may note that, since the Lewis number for ion diffusion is far from unity, some difference in heat transfer should be found in a frozen boundary layer even if recombination is completed at the surface. In addition, the questions of the electron temperature and the effect of inelastic collisions have to be clarified.

There are limited amounts of heat transfer measurements made in shock tubes for total enthalpies corresponding roughly to re-entry from lunar mission.<sup>143, 147</sup> Disagreement, however, exists which has yet to be resolved. (Radiative heat transfer can be an important factor.) Perhaps the more basic problem in this area is that of the N-N<sup>+</sup> collision cross section, on which experimental data pertaining to the temperature range of interest are not available.

### Ablation and Related Studies

The impetus of the missile and manned-satellite programs has led to tremendous progress in the ablation field in the past few years. Considerable understanding has been gained on the theory of melting and subliming ablations, as summarized in the Adams' review.<sup>148</sup> Recent development of ablation as a heat alleviation scheme has greatly enlarged the scope of the previous study, as new materials and concepts are explored.

According to the surface materials, ablations may be classified into four categories: 1) materials such as quartz which melt and then vaporize, 2) plastics such as teflon which "depolymerize" to a gas but do not melt, 3) composite materials, such as phenolic resin reinforced with nylon, which "pyrolyze" and form char layer, and 4) materials such as graphite which ablate by oxidation. The earlier work summarized in Adams' paper concerns mostly the first two categories, for which there is reasonable agreement between theoretical prediction of heats of ablation and experimental data.<sup>148, 149</sup> The third and fourth classes of materials represent the more complicated and less understood areas in this field. The state of the art of the ablating heat shield in all four categories has been examined recently by Steg and Lew.<sup>150</sup>

The charring plastics belonging to the third class are attractive but present the most complicated picture. Although the heat of ablation may be estimated reasonably with the aid of experiment,<sup>148, 150</sup> the behavior of the char layer and the effectiveness of the scheme are not understood easily. The problem involves chemical reactions in depth, and mass and energy transfer in porous media, among other factors. One of the studies of this sort has been made recently by Scala and Gilbert<sup>151</sup> in which a detailed analysis is given for a hydrocarbon plastic at the stagnation point. The analysis leads to the thermal and mechanical stress consideration that predicts fracture and removal (spallation) of the char. As pointed out by Steg and Lew,<sup>150</sup> however,

the theories are heavily dependent on a number of parameters that have to be determined experimentally. In addition, it is apparent that the forming and fracture of char may effect a transition to the turbulent boundary layer, as well as introduce unsteadiness in the outer flow which further complicates the problem.

The use of graphite as a thermal shield is limited by the heat penetration, which gives rise to a high back-face temperature. Recent interest in its application lies presumably in the recognition of the high degree of anisotropy in the crystal structure of pyrolytic graphite. By proper orientation, the thermal conductivity can be reduced by more than a hundredfold. The overall performance of a system using pyrolytic graphite for lifting re-entry has been analyzed recently by Noland and Scala,<sup>152</sup> which indicates outstanding qualities and low oxidation rate. The oxidation kinetics of this material have been studied by Horton.<sup>153</sup> Combustion of a carbon surface exposed to an air stream is considered by Moore and Zlotnick<sup>154</sup> for surface temperature 1000° to 3000°K. This study adopts the surface oxidation kinetics of Blyholder and Eyring<sup>155</sup> and employs a hypothetical reaction surface to treat gas-phase combustion. Nevertheless, our knowledge of carbon combustion is still far from being certain (in spite of man's long association with coal), and important data on graphite ablation may have to come from simulated experiments. As Steg and Lew observed,<sup>150</sup> test results from most existing facilities may not be reliable on account of contamination affecting the chemistry.

Considering ablation as a self-regulating mass transfer system, much of our knowledge and experience gained from study of transpiration cooling should be helpful.<sup>156, 157</sup> From this viewpoint, the effect of the molecular weight of the ablating vapor on laminar heat transfer was examined recently by Faulders.<sup>158</sup> It is of interest to note that the amount of air as coolant required by a system using the forced mass-transfer technique actually is smaller than that required by ablation<sup>150</sup>; its feasibility remains to be demonstrated, however.

### VII. Radiation from High-Temperature Air

Radiative heating constitutes a serious problem<sup>2, 3, 5, 8</sup> for the re-entry of a nonlifting vehicle at supersatellite speeds. For a lifting vehicle that slows down at higher altitude, radiative heating still may remain critical because of the higher temperature level due to chemical nonequilibrium. Further interest in air radiation results from its importance in the observation of, or from, re-entering vehicles. In the following discussion, the present knowledge of air radiation is summarized, and applications to blunt-body flows are discussed.

#### Equilibrium Radiation

Theoretical and experimental work by Keck, Kivel, Meyerott, Wurster, and others<sup>106, 159-162</sup> over the past years provides valuable information on air radiation from 3000° to 12,000°K. They have identified the major radiating species, determined the spectral distributions, and provided formulas for scaling these data to higher temperatures. Treanor<sup>163</sup> has examined recently the data for spectral distributions in the range 3000° to 8000°K. Although the gross aspects are understood well, substantial discrepancies exist for several molecular bands, notably the N<sub>2</sub>(1+) and N<sub>2</sub><sup>+</sup>(1-) systems. One also may note the uncertainty in the level of Kramer's radiation<sup>164</sup> which results from collisions of electrons and atomic ions and is important above 12,000°K.

Lees<sup>165</sup> previously has given an empirical fit for the equilibrium emissivity of air as computed by Kivel and Bailey<sup>166</sup> in the range 8000° to 12,000°K. Thomas<sup>167</sup> provides a new fit based on the more recent data of Meyerott and co-workers.<sup>162</sup> He applies it to estimate radiant heat transfer

at the stagnation point of an ellipsoid. Strack<sup>168</sup> integrates the heat transfer around a sphere for a constant-density flow model. The modification of the Rankine-Hugoniot shock relations by the equilibrium radiation field at high temperature (including electromagnetic forces) is studied by Pai.<sup>169</sup> In many applications, frequency-averaged absorption coefficients are used to compute heat transfer. This procedure could be inaccurate below 8000°K. Strack<sup>170</sup> gives a correction in terms of average absorptivity. However, this correction and the estimates of heat transfer are subject to the same uncertainties as the basic radiation data. The similarity parameters in the radiation flow fields are discussed by Penner and co-workers<sup>171</sup> and by Goulard.<sup>172</sup>

#### Nonequilibrium Radiation behind a Normal Shock

Translational and vibrational temperatures behind a normal shock at satellite or higher speeds are much higher than the equilibrium value because of nonequilibrium processes. Since the radiation intensity tends to follow the translational and vibrational temperatures, strong radiation overshoots may be expected above the equilibrium level.

The basic data for equilibrium radiation<sup>166</sup> can be applied to study the "nonequilibrium radiation." The radiation temperature may be taken equal to the translational temperature. As discussed by Tears and co-workers,<sup>173</sup> this assumption is conservative (too high) because the electronic excitation may not follow readily the translational temperature. For instance, below 3000°K, it has been shown<sup>174</sup> that electronic excitation of atoms occurs preferentially through collisions with vibrationally excited molecules. This mechanism essentially relates the radiation temperature to a vibrational temperature. Recent photometric measurements of radiation behind strong shocks in nitrogen appear to substantiate this interpretation.<sup>117</sup> However, this model does not provide good correlations for air. One may note that the coupling process between vibration and dissociation which controls the vibrational temperature still requires further study (see Sec. V).

Shock-tube experiments<sup>173</sup> have verified the binary-scaling law near the shock front, and the law can be used to correlate the radiation profiles behind the shock at different density levels. On account of the binary nature of the excitation processes, the intensity of the nonequilibrium radiation and the corresponding relaxation time are, respectively, proportional and inversely proportional to freestream density. Hence, the total nonequilibrium radiation is independent of density level (radiation plateau).

This forementioned behavior holds only when collisions are sufficiently frequent to maintain equilibrium populations in the excited states corresponding to the effective temperature of the excitation process. At lower densities, however, radiative emission may provide a faster path for de-excitation, thus depleting the population of the excited species and reducing the radiation intensity from its equilibrium value. Such "collision-limiting effect" has been observed below 3000°K for shocks in inert gases.<sup>175</sup> So far, it has not been clearly identified for strong shocks in air. In the range of collision-limiting, binary scaling would not apply (unimolecular process). Presumably, collision-limiting is not expected<sup>164</sup> to be of significance for altitudes below 200,000 ft.

#### Radiation from Nonequilibrium Blunt-Body Flows

In addition to the basic radiation data and knowledge of the processes for electronic excitations, calculations of the nonequilibrium radiation field around blunt bodies require a detailed knowledge of the nonequilibrium flow field. To date, a few general features of the blunt-body field have been recognized.

The radiation field of the nonequilibrium air flow around a hemisphere has been calculated by Rose and Teare<sup>176</sup> for a

flight speed of 35,000 fps, an altitude of 200,000 ft, and a 1-ft nose radius. They apply a streamtube method and assume local vibrational equilibrium. To account for the finite rise time of the radiation intensity (due to adjustment of the electronic states), a linear rise of radiation intensity is assumed. The slope of this linear rise along a streamline is adjusted so that the "peak" of the approximate radiation profile corresponds to that observed in experiment for a normal shock.<sup>173</sup>

It may be pointed out that the radiation field around the re-entry vehicle perhaps may be more simply and consistently described by mapping the radiation profiles behind normal shocks along the streamline according to the theory of Gibson and Marrone<sup>169</sup> discussed in Sec. V.

#### Influence on Configuration Design

Although the radiation heating cannot be predicted accurately at the present time, its influence on the configuration design of vehicles with high re-entry speeds may be anticipated. As noted earlier by Dorrance and many others, radiative heating can be reduced by decreasing the nose radius, since at the stagnation point, the equilibrium radiative heat flux is roughly (for an optically thin gas) proportional to the thickness of the gas cap and thus the nose radius  $R$ .<sup>177</sup> This will, of course, raise the laminar convective heating rate that is inversely proportional to  $R^{1/2}$ . A minimum total heat transfer rate can be attained where the radiative heat flux is half as large as the convective. This conclusion also holds with nonequilibrium radiation, since the latter may be regarded as being independent of  $R$ . Recent discussion by Mager<sup>178</sup> shows that such optimization may result in substantial heat flux reduction whenever the radiation becomes the same order or larger than the convection. The radiative heating consideration thus suggests use of small nose bluntness for superorbital re-entry. In the density and temperature ranges of interest, the total emissivity increases rapidly with temperature, and radiative heat flux therefore will increase even more rapidly with temperature. Since the shock angle controls the temperature, it would appear that re-entry from very high superorbital speed, if ever possible, requires not only small bluntness but also slender shape.<sup>3</sup>

#### Radiation Energy Loss

At very high re-entry speed, the energy loss by emission may be sufficiently large to affect the flowfield. An estimate of the condition under which this effect becomes significant has been made by Goulard<sup>9</sup> for equilibrium air (see also Fig. 1). Tears and co-workers<sup>173</sup> have estimated this effect in the nonequilibrium zone and find some effect for an altitude of 220,000 ft at a speed of 35,000 fps.

For return from Mars and other planets corresponding to higher re-entry speed, a sizable fraction of the flow energy in the shock layer will be radiated away. As flow speed is increased, the principal radiation of the gas-cap energy moves to shorter and shorter wavelengths so that it can be absorbed more readily in the air upstream. As observed by Allen,<sup>3</sup> because of the "trapping" of radiation energy immediately upstream of the shock, the body must now accept more than half of the energy radiated from the gas cap. In the extreme case, the effect may be considered as a sort of shock thickening that is characterized by the balance of convective steepening and of the dissipative action of the radiative heat transfer. A model of this radiation-resisted shock was analyzed recently by Clarke<sup>179</sup> and also by Yoshikawa and Chapman.<sup>180</sup>

#### Precursor Electrons

Since air (oxygen molecule) can be ionized readily by ultraviolet radiation, the radiation trapping discussed

previously must involve a significant degree of photoionization. Even for shock strength corresponding to satellite or lower speed, the weak ionization by ultraviolet radiation from behind the shock may be detectable and has been reported.<sup>181, 182</sup> Of course, the observed precursor effects in certain cases may be attributed to electrons diffusing ahead of the shock front.<sup>183, 184</sup> Further study clearly is required to separate the causes of, and identify the domains of, the precursor effects. It may be mentioned, nevertheless, that the photoionization by precursor radiation has, in fact, been used recently by Lin<sup>185</sup> to explain and correlate the "leading-edge echoes" observed in radar returns from manned satellites and meteors during entry.

### VIII. Concluding Remarks

Considerable advances in hypersonics have been achieved during the past year. Theoretical and experimental studies have yielded many valuable data for high-speed re-entries and have identified, in the meantime, a few specific problems whose solution is essential for future development. Such a general impression should be evident from this survey, particularly in the categories of wakes, rarefied-gas flows, nonequilibrium flows, mass and heat transfer, and high-temperature air radiation.

In concluding this work, a reflection on certain problem areas that remain to be further explored perhaps may be desirable. In doing so, only some of the most basic problems will be mentioned.

In the category of boundary layers and wake flows, the problem requiring most attention is perhaps the transition from laminar to turbulent flow. The transition mechanism for the hypersonic wake has yet to be understood. The empirical nature of the existing analyses of the near wakes and the base-flow regions has left much to be desired. In the area of rarefied gas flows, one may note that the usefulness of the free-molecule flow theory will depend on our knowledge of the surface accommodation coefficients. For a meaningful determination of these coefficients, one must wait for the development of experimental techniques to generate molecule beams in the range of 1 to 10 ev. Our knowledge of transition from the free-molecule to continuum flow regime has been limited only to very special flow situations, such as the Couette flow and the transition zone of a normal shock, mainly because of the analytical difficulty in handling the Boltzmann equation. The apparent success of the recent treatment of the shock transition zone by Leipman and co-workers, using Krook's kinetic model, seems to warrant application of the same model to other flow situations. The continuum approach based on the Navier-Stokes equations also may be extended to treat rarefied-gas problems in other than the stagnation region and to provide important information, such as the lift to drag ratio and total heat transfer value, for lifting re-entry analyses.

Most current analyses of nonequilibrium air flow with coupled processes concern nozzle and blunt-body flows. For the inviscid and viscous (laminar) wake flows, there are no corresponding analyses. The analytical problem for both inviscid and viscous wakes actually are relatively simple. There is always the question of whether the available rate data are accurate enough to warrant a detailed analysis of the coupled reactions. Our uncertainty about the rate constants used may not be too critical in most of the dominant processes because of the nonlinear nature of the kinetics per se. However, in certain processes such as those governing electrons and ions, the magnitude of the error in the solution is in direct proportion to the error in the rate constant. In this regard, one may recall the problems that remain with the rates of the coupled processes of vibration and dissociation, which are important in interpreting and correlating kinetic data of dissociation.

Similarly, the question arises regarding the degree of uncertainty in transport coefficients of the various gas species at high temperatures when the coupled air chemistry is considered in the boundary layer. When ionization occurs, there is further uncertainty because of the lack of basic information on the N-N<sup>+</sup> collision cross section that controls the ion diffusion process. This cross section has yet to be determined experimentally. The study of the ionization in boundary layers cannot be regarded as being complete until the nonequilibrium aspect of the problem is treated adequately. As speed increases, the surface catalycity for recombination of atoms (as well as ions) assumes more and more importance at high altitudes. The ability to control and predict convective heating will depend to a considerable extent on our knowledge of the surface catalysis. Available information on surface catalycity is far from sufficient.

From the viewpoint of application, one disturbing fact about the high-temperature air radiation is that there are sizable discrepancies among available data on the absorption coefficients for a number of important band systems and other sources. Without resolving these discrepancies, our knowledge of the radiation intensity, for both equilibrium and nonequilibrium radiation, can be subject to an uncertainty as much as a factor of 2 or more. For radiation with chemical nonequilibrium, the processes leading to electronic excitation determine the peak intensity and the radiation profile behind the shock. The controlling process for electronic excitation in air in the temperature range of interest has not been identified. It may be pointed out in this connection that lack of equilibrium in the electronic states also will reduce considerably the equilibrium constant and thus the rate of dissociation for nitrogen at satellite and higher speeds.

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## Measurement of Stream Velocity in an Arc

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In electric arcs, there generally exists a considerable streaming of plasma caused by Lorentz forces generated by self-magnetic fields. In this report, the method of measurement of such streaming velocity is described. The arc used is a direct current thermal arc of 105 amp in argon at atmospheric pressure. At first the temperature of the arc was measured by a spectroscopic method and by a thermocouple, and a temperature distribution ranging from 16,000° to 1500°K was obtained. The streaming velocity was evaluated by measurement of the drag of a small plate swept across the arc, using the result of the temperature measurement, the drag coefficient of the plate, and the calculated density and viscosity coefficient of the plasma. At a section 2 mm from the cathode, the evaluated stream velocity was 135 m/sec at the center and 25 m/sec at the edge of the arc column, from cathode to anode. The error in measured temperature is estimated at less than 10%, and the error in the measured stream velocity is considered to be less than 17.5%, excluding the error due to some change of the arc discharge caused by inserting the drag measuring plate.

### Introduction

INERT gas thermal arcs are used widely today as sources of high-temperature gas plasmas. However, they generally involve a considerable streaming of plasma (from cathode to anode) caused by Lorentz forces generated by the self-magnetic field. Knowledge of this streaming velocity is important to the understanding of the mechanism of arc discharge or to the use of it as a plasma source. Wienecke<sup>1</sup> evaluated the stream velocity in an arc from a high-speed motion picture record of the arc which was re-initiated after the current was stopped momentarily. Reed<sup>2</sup> obtained the stream velocity in the neighborhood of the anode of an arc by measuring total pressure through a small hole on the anode plate. In this report, a method of determination of the stream velocity in an arc by measuring the drag of a small plate, which is swept across the arc, is described. In this method it is necessary to know the temperature of the arc, and so at first a temperature measurement was made.

### The Arc

The arc used is a direct current thermal arc in argon at atmospheric pressure, burning between a 3-mm-diam tungsten

cathode formed into conical tip and a 20-mm-diam copper plate anode. The electrodes are arranged vertically in an arc chamber (14 cm in height and 11 cm square in cross section) and are cooled by water of fixed flow rate. In this case, the arc had axial symmetry.

In this experiment, argon of less than 0.01% impurity was bled through the chamber at a flow rate of 9.5 liter/min. The arc current was kept at 105 amp and the distance between electrodes at 5.8 mm, whereas the potential difference between them was maintained at 13.7 v. The arc discharge was stable for long-time operation, and consumption of electrode materials was unnoticed.

### Temperature Measurement

The temperature in the arc column was measured spectroscopically by the method developed by Larentz<sup>3</sup> and Olsen,<sup>4</sup> and the temperature at the outside of it was measured by a thermocouple.

### Measurement of Spectral Line Intensities

Spectral line intensities were measured with a spectrograph having a dispersion of 1/150 mm/Å in the infrared and a photometer consisting of a vacuum-type photoelectric cell and DuBridge circuit. The linearity of the intensity detection system was checked by changing the area of the slit of the spectrograph when lighted uniformly. The arc image was focused

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